The RR interval spectrum, the ECG signal and aliasing

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Abstract

We discuss the relationship between the RR interval spectral analysis and the spectral analysis of the corresponding ECG signal from which the RR intervals were evaluated. The ECG signal spectrum is bounded below the frequency f_B by using an electronic filter and sampled at rate larger than $2f_B$, thus excluding aliasing from spectral analysis. A similar procedure cannot be applied to the RR interval spectral analysis, and in this case aliasing is possible. One of our main effort in this paper is devoted to the problem of how to detect aliasing in the heart rate spectral analysis. In order to get an insight we performed an experiment with an adult man, in which the ECG signal was detected in a case where the breathing rate was larger than half the heart rate. A constant breathing rate for time intervals exceeding 5 minutes was monitored with good accuracy using a special breathing procedure. The results show distinctively a very sharp peak in the spectral analysis of the ECG signal and corresponding (diffused) aliasing peaks in the RR interval spectral analysis.

New method of dealing with unevenly sampled data was developed which has interesting anti-aliasing properties. There are indications that the VLF peaks of the RR spectrum are originated by aliasing. Some of the LF peaks may have the same property.

Keywords: Hart rate, ECG signal, Spectral analysis, Aliasing

I. INTRODUCTION

The R-R interval spectral analysis is usually based on heart rate data collected in two ways. In one method the data are collected by analog to digital conversion of the ECG signal and computer evaluation of the R-R intervals from the ECG signal. In the second method, devices are used whose output is the R-R interval alone. The advantage of the first

method is the control of accuracy and flexibility of the evaluations. The second method has the advantage of storing smaller amount of data, and it can be easily used on-line.

In the first method, usually the number of collected data (sampled ECG signal) is of two to three orders of magnitude larger than the R-R interval data. Thus if only R-R interval is analyzed a large amount of data is unused. In this paper we are trying to take advantage of the ECG sampled signal and to derive new information in addition to the conventional R-R interval analysis [1],, [2], [3], [4].

The ECG signal spectrum is bounded below the frequency f_B by using an electronic filter and sampled at rate larger than $2f_B$, thus excluding aliasing from spectral analysis. [5] A similar procedure cannot be applied to the R-R interval spectral analysis, and in this case an aliasing is possible. One of our main efforts in this paper is devoted to the problem of how to detect aliasing in the R-R interval spectral analysis.

In order to get an insight, we performed an experiment, in which the ECG signal of one of the authors (A.G) was detected while the breathing rate was larger than half the heart rate. A constant breathing rate for a time exceeding 5 minutes was monitored with good accuracy using a special breathing procedure with a metronome. The results show distinctively a very sharp peak in the spectral analysis of the ECG signal and corresponding (diffused) aliasing peaks in the R-R interval spectral analysis.

The spectral analysis of the ECG signal was performed with the standard FFT procedures. The spectral analysis of the R-R intervals was performed with several techniques in order to take into consideration that the data were unevenly sampled. This is presented in section 2. In section 3 we discuss the possibility of aliasing in the spectral analysis of the R-R intervals. In section 4 we compare power estimations of ECG's and R-R intervals of 3 experiments. In section 5 we analyze the results. In section 6 summary and conclusions are presented.

II. SPECTRAL ANALYSIS OF UNEVENLY SAMPLED DATA

The methods of spectral analysis are well developed for evenly sampled data [5], [6]. The R-R interval data are unevenly sampled in time. In most cases an analysis is performed with respect to beat numbers which are evenly spaced. We will below justify this method using least squares principles. But as was recently indicated by Laguna et Al. [7], the resampling of data is causing the appearance of additional harmonics. They recommend to use a method developed by N. R. Lomb [8]. The errors of resampling the beats can, to large extent, be overcome by using a cubic spline interpolation. In this work we are suggesting a new method of treating unevenly sampled data, which, unexpectedly, gave good results beyond the Nyquist frequency.

A. Analysis according to beat numbers

Let us assume that the RR intervals are given at unevenly sampled times t_n, with the values $s\left(t_n\right)$, where n is the beat number, $n = 1 \cdots N$. Let us divide the interval $[t_1, t_N]$ into equal subintervals

$$\Delta\tau = \frac{t_N - t_1}{N-1}, \tag{1}$$

and let us generate in the interval $[t_1, t_N]$ evenly sampled times:

$$\tau_n = (n-1)\Delta\tau + t_1. \tag{2}$$

We will use the discrete time Fourier transform (DFT) for a basis formed from the evenly sampled times τ_n. We will assume that

$$s(t_n) = \frac{1}{N}\sum_{k=1}^{N} S_k \exp\left(i\omega_k\tau_n\right), \qquad \omega_k = 2\pi\left(k-1\right)/\left(N\Delta\tau\right). \tag{3}$$

The coefficients S_k will be determined by minimizing the expression

$$\sigma = \sum_{n=1}^{N}\left\{\left[s(t_n) - \frac{1}{N}\sum_{k=1}^{N} S_k \exp\left(i\omega_k\tau_n\right)\right]\left[s(t_n) - \frac{1}{N}\sum_{k=1}^{N} S_k^* \exp\left(-i\omega_k\tau_n\right)\right]\right\} \tag{4}$$

with the result

$$S_k = \sum_{n=1}^{N} s\left(t_n\right) \exp\left(-i\omega_k \tau_n\right). \tag{5}$$

Eqns. 5 and 3 can be handled easily with standard FFT programs. This is the usual procedure which is adopted in most of the papers dealing with R-R interval analysis. [3] [4]

B. Other Methods

FFT can be applied more efficiently if the unevenly sampled data are interpolated at evenly spaced intervals of Eq. 2. The cubic spline interpolation is one of the good ways to do it.

The Lomb method [8] was extensively analyzed in ref. [7]. We give here only the formulae in the form of the Lomb normalized periodogram

$$P_X\left(\omega_k\right) = \frac{1}{2\sigma^2}\left\{\frac{\left[\sum_{n=1}^{N}\left[s\left(t_n\right)-\bar{s}\right]\cos\left(\omega_k\left(t_n-\tau\right)\right)\right]^2}{\sum_{n=1}^{N}\cos^2\left(\omega_k\left(t_n-\tau\right)\right)} + \frac{\left[\sum_{n=1}^{N}\left[s\left(t_n\right)-\bar{s}\right]\sin\left(\omega_k\left(t_n-\tau\right)\right)\right]^2}{\sum_{n=1}^{N}\sin^2\left(\omega_k\left(t_n-\tau\right)\right)}\right\} \tag{6}$$

where $\bar{s}$ and σ^2 are the mean and variance of the data and the value of τ is defined as

$$\tan\left(2\omega_k\tau\right) = \frac{\sum_{n=1}^{N}\sin\left(2\omega_k t_n\right)}{\sum_{n=1}^{N}\cos\left(2\omega_k t_n\right)} \tag{7}$$

C. Non-Uniform Discrete Fourier Transform (NUDFT)

We present here a new method of treating unevenly spaced events which we call the "non-uniform discrete Fourier transform" (NUDFT).

Let us assume that $s\left(\tau_n\right)$ are the exact values of the signal at the points given by Eq. 2. The corresponding DFT is

$$S_k = \sum_{n=1}^{N} s\left(\tau_n\right) \exp\left(-i\omega_k \tau_n\right). \tag{8}$$

Our aim is to find a good approximation to this expression in terms of the unevenly sampled signal $s(t_n)$.

We start with the Euler summation formula

$$\sum_{n=1}^{N} f(\tau_n) = \frac{1}{\Delta\tau}\int_{\tau_1}^{\tau_N} f(\tau)\,d\tau + \frac{1}{2}\left[f(\tau_1) + f(\tau_N)\right] + \frac{\Delta\tau}{12}\left[f'(\tau_N) - f'(\tau_1)\right] + O(\Delta\tau^2) \quad (9)$$

and make the following decomposition of the integral on the right hand side of Eq.9:

$$\int_{\tau_1}^{\tau_N} f(\tau)\,d\tau = \int_{t_1}^{t_2} f(\tau)\,d\tau + \int_{t_2}^{t_3} f(\tau)\,d\tau + \cdots + \int_{t_{N-1}}^{t_N} f(\tau)\,d\tau \quad (10)$$

and approximate each of the integrals on the right hand side with the trapezoidal rule:

$$\int_{\tau_1}^{\tau_N} f(\tau)\,d\tau = \frac{1}{2}\left[f(t_1) + f(t_2)\right](t_2 - t_1) + \cdots + \frac{1}{2}\left[f(t_{N-1}) + f(t_N)\right](t_N - t_{N-1}) + O(\Delta\tau) \quad (11)$$

From Eqs. 9 and 11 we obtain:

$$\sum_{n=1}^{N} f(\tau_n) = \frac{1}{2\Delta\tau}\left\{\left[f(t_1) + f(t_2)\right](t_2 - t_1) + \cdots + \left[f(t_{N-1}) + f(t_N)\right](t_N - t_{N-1})\right\}$$

$$+ \frac{1}{2}\left[f(t_1) + f(t_N)\right] + O(\Delta\tau)\,. \quad (12)$$

When the t_n are equally spaced Eq. 12 becomes an identity with the $O(\Delta\tau) = 0$, therefore it seems to us that Eq. 12 is satisfied with an higher accuracy than just $O(\Delta\tau)$. Eq. 12 can be applied to approximate Eq. 8 with the substitution

$$f(t_n) = s(t_n)\exp(-i\omega_k t_n)\,, \quad (13)$$

and the final result, the approximation to Eq. 8, after rearranging the terms, becomes:

$$S_k = \sum_{n=1}^{N} c_n s(t_n)\exp(-i\omega_k t_n) + O(\Delta\tau)\,, \quad (14)$$

where

$$
\begin{aligned}
c_1 &= \tfrac{\Delta\tau + t_2 - t_1}{2\Delta\tau}, \\
c_2 &= \tfrac{t_3 - t_1}{2\Delta\tau}, \\
&\vdots \\
c_{N-1} &= \tfrac{t_N - t_{N-2}}{2\Delta\tau}, \\
c_N &= \tfrac{\Delta\tau + t_N - t_{N-1}}{2\Delta\tau},
\end{aligned}
\tag{15}
$$

with the inverse formula

$$
s(\tau_n) = \frac{1}{N}\sum_{k=1}^{N} S_k \exp\left(i\omega_k \tau_n\right) + O\left(\Delta\tau\right), \qquad \omega_k = 2\pi\left(k-1\right)/\left(N\Delta\tau\right), \tag{16}
$$

which is an interpolation formula for $s\left(t_n\right)$ at the evenly spaced points $\tau_1 \cdots \tau_N$.

III. ALIASING

Aliasing is a result of undersampling and is a well known phenomenon. In ref. [9] aliasing was looked upon from the point of view of symmetry. It is an example of wrong symmetry, and as such should be given more attention. It is the outcome of an incomplete basis. It was found in ref. [9] , that for evenly sampled data with a sampling rate f_S, the spectral amplitude $S(f)$ evaluated with FFT, has the following symmetry properties

$$
\left|S\left(f\right)\right| = \left|S\left(f \pm f_S\right)\right| = \left|S\left(-f \pm f_S\right)\right| = \left|S\left(\pm f \pm n f_S\right)\right|, \tag{17}
$$

where f is the frequency and n is an arbitrary integer.

In order to avoid the aliasing symmetry of Eq. 17, the frequencies should be bounded by the Nyquist frequency (denoted here by f_B) according to

$$
f_B = \frac{f_S}{2}. \tag{18}
$$

The ECG signal was sampled with sampling rate 250 Hz, and an electronic filter was applied, which have eliminated practically all frequencies above 32 Hz, thus aliasing can not occur at frequencies below 125 Hz or even below 32 Hz. The R-R intervals were calculated directly from the ECG signal. The sampling rate for R-R intervals can be defined only for

evenly sampled data, for the methods which interpolates the unevenly sampled data, or one can consider the average sampling rate from Eq. 1, in both cases

$$\bar{f_S} = 1/\Delta\tau = 2f_N, \tag{19}$$

where f_N is the Nyquist frequency for the R-R intervals. As the ECG signal contains frequencies much grater than f_N, and the R-R intervals are derived from the ECG signal, one can not be sure that the spectral analysis of the R-R intervals is free from aliasing. As a matter of fact there are indications of aliasing in some rare cases. [10] [11] [12] [13] [14] [15] One way to identify aliasing is to change the sampling rate and follow the changes in the spectrum. Unfortunately, for the R-R intervals, one can not speak about a definite sampling rate, but rather can consider a distribution of sampling rates. The changes in sampling rate required to observe aliasing are of the same order as the fluctuations in the sampling rate. Therefore in practice it is almost impossible to observe consistent changes in the spectrum slightly changing the heart rate.

Other possibility of detecting aliasing is by comparing the heart rate spectrum with the ECG signal spectrum. Marked differences below the Nyquist frequency for the power distribution of the RR intervals compared to the ECG signal power distribution in the same range may indicate aliasing. But we do not have yet a sound basis to treat this problem.

We have devised an experiment which definitely demonstrates the aliasing in the R-R intervals spectrum. To the best of our knowledge this is the first experiment in which one can exactly know the correct frequency above the Nyquist frequency and can follow the development of the aliasing, which appears to be diffused to great extent because the symmetry of Eq. 17 is represented not by one sampling rate but by a distribution of sampling rates, as the R-R interval is unevenly sampled.

Below we describe 3 experiments. One of them was devised to demonstrate aliasing and the other two for learning about the relations between the R-R interval spectrum and the spectrum of the ECG signal.

IV. THREE EXPERIMENTS

We present below results of three experiments. In the first experiment the ECG signal was collected in a normal resting state. The aim of this experiment was to compare the ECG spectrum with the R-R intervals spectrum. In the second experiment very slow breathing was monitored at a rate of 0.04 Hz. Again the ECG and R-R interval spectra were compared. In the third experiment very fast breathing was accurately monitored at the rate of 74/min and 84/min. These respiratory rates were above half of the heart rates thus allowing to observe in detail the development of aliasing.

A. The First Experiment

In this experiment (linked with the names of Zahi and Ori, where the second is one of the authors: O.G) which was done in normal, resting conditions, we compare the power estimation of the R-R interval and the ECG signal, from which the R-R interval was obtained. The ECG signal was sampled at a rate of 250 Hz. Stable intervals of 7 minutes duration were chosen for analysis.

In Fig. 1a the power distribution of the ECG signal of Zahi is depicted. The attenuation of the power with increasing frequency above 12 Hz is due to the action of an electronic filter. Above 32 Hz the contribution is practically zero. The average heart rate was 0.97 Hz. The above results were zoomed to the interval [0-12] Hz in Fig. 1b. One can see distinctively the peak around the average heart rate and the higher harmonics of this peak. The second harmonic is missing, but the third, fourth, fifth and sixth are distinctively visible, higher harmonics became more and more smeared and indistinguishable above the sixth harmonic. One should also note the large difference in power in the heart rate range, below the Nyquist frequency of 0.49Hz, which is much smaller compared to the peak around the average heart rate (0.97 Hz).

The power distribution of the RR intervals in the range [0-0.5] Hz was computed ac-

cording to the methods discussed in section 2 and are presented in Figs. 2a (DFT, beat number analysis), 2b (Spline interpolation), 2c (NUDFT). For comparison also the power distribution of the ECG signal in the above range is presented in Fig. 2d.

The results of Figs. 2a, 2b, and 2c are quite similar, but the spline interpolation (Fig. 2b) and the NUDFT (Fig. 2c) are practically identical. The three graphs show the structure commonly found in the power estimation analysis of RR intervals, namely the existence of the "high frequency" (HF), "low frequency" (LF) and the "very low frequency" (VLF) peaks. The ECG spectrum shows qualitatively the same structure (but not a quantitative agreement), except that the ECG spectrum is highly suppressed below 0.04 Hz, in the VLF region, indicating a possibility of aliasing in this region in the RR analysis.

In Figs. 3 and 4a-4d the results of Ori are presented. The conclusions are similar to those of Zahi, except that in the ECG spectrum both VLF and LF peaks are missing, indicating the possibility of aliasing in these regions for the RR analysis. Also in the ECG spectrum of Ofek, Fig. 5b the VLF and LF, present in Fig. 5a, are missing. VLF is missing in J.C.'s ECG spectrum (see Figs. 6a-6b).

B. The Second Experiment

In this experiment (linked again with the name Ori) we have checked the ECG spectrum near the VLF region, as the VLF was absent in the ECG spectrum for the resting state in the first experiment. The question was whether such a result persists in all ECG spectra. Therefore we have probed the VLF region by monitoring very prolonged breathing with a rate of 0.04 Hz. For the spectrum of RR intervals we found that the DFT, Spline interpolation and NUDFT gives similar results, and again NUDFT was practically identical to the spline interpolation. Therefore we present only the results of NUDFT, which are presented in Fig. 7a. For comparison the spectrum of the ECG signal is given in Fig. 7b. In Fig. 7a one can see a very clean pattern of a peak at 0.04 Hz and its higher harmonics. In Fig. 7b one can see a similar but somewhat diffused pattern. Thus this experiment indicates that

similar respiratory patterns exists in both the RR as well as in the ECG signal.

C. The Third Experiment

In this experiment (linked to the name Alex, who is one of the authors: A.G) very fast breathing was accurately monitored at the rate of 74/min and 84/min respectively. These rates were well above half of the average heart rate thus allowing to observe in detail the development of aliasing. In Fig. 8 the ECG spectrum is dominated by the very high and narrow peak at the frequency $f_1 = 1.234Hz$, also its higher harmonics can be distinctively seen. The frequency f_1 is just the breathing frequency 74/min. In the same figure one can also see the diffused peaks near the average heart rate frequency of 1.636 Hz and its higher harmonics. One should observe aliasing at about $1.636 - f_1 = 0.402Hz$. Indeed one can see diffused peaks around that frequency in Fig. 9a, which displays the power estimation of the RR intervals using the NUDFT (which below the Nyquist rate is similar to the spline interpolation). The width of this region can be estimated by noting that the RR intervals have different instantaneous sampling rates which are equal to the inverse of the RR interval time. In Fig. 10 we have calculated the distribution of the sampling rates by dividing the frequency region into 100 beans. We have shifted that distribution by subtracting f_1. As one can see the results are confined approximately to the region 0.32-0.47 Hz. Indeed the aliasing peaks of Fig. 9a appear in this region. The pictures below the Nyquist frequency are very similar for the DFT, NUDFT, the spline interpolation and the Lomb method (Fig. 9b) with a similar aliasing behavior.

In principle the NUDFT and the Lomb methods should not be used above the Nyquist frequency. Surprisingly enough we have found that both methods have a sharp peak at f_1, as can be seen in Figs. 9a and 9b. Both methods do not have the aliasing symmetry of the DFT as given by Eq. 17, therefore the results are not symmetric with respect to the Nyquist frequency (half the sampling rate), as it is satisfied, for example, in the case of the spline interpolation. We have found an exact result at f_1 and a diffused aliasing around 0.4 Hz. It

is interesting to note that both methods give almost the same result below and above the Nyquist frequency. One can interpret the appearance of the sharp peak at f_1 as a result of a partial destruction of aliasing symmetry due to uneven samplings.

Similar results for the breathing frequency 84/min are presented in Figs. 11-12.

V. SUMMARY AND CONCLUSIONS

The ECG signal spectrum is bounded below the Nyquist frequency f_B by using an electronic filter and sampled at rate larger than $2f_B$, thus excluding aliasing from spectral analysis. A similar procedure cannot be applied to the RR interval spectral analysis, and in this case an aliasing is possible. One of our main efforts in this paper was devoted to the problem of how to detect aliasing in the R-R interval spectral analysis.

In order to get insight into this problem three experiments have been analyzed. In the first experiment the ECG signal was collected in a normal resting state. The aim of this experiment was to compare the ECG spectrum with the R-R interval spectrum. In the second experiment very slow breathing was monitored at a rate of 0.04 Hz. Again the ECG and R-R interval spectra were compared. In the third experiment very fast breathing was accurately monitored at the rate of 74/min and 84/min respectively. These respiratory rates were above half of the heart rates thus allowing to observe in detail the development of aliasing.

The experiments which were described above led us to the following conclusions:

1. The spectral analysis of the ECG signal is more sensitive and accurate compared to the R-R interval spectral analysis and is free from aliasing. Still in the present stage it contains too much information to be of practical use. Efforts should be made to understand what will be the best way to extract information (not related to the heart condition alone as in the standard analysis of ECG) about the external influences on the heart signal.

2. We have conducted an experiment which gave a clear insight about the mechanism of aliasing in the R-R interval spectrum. The very sharp peak in the spectrum of the ECG signal, which came as the result of enforced quick breathing, reappeared as a diffused signal in the RR spectrum. The extension of the diffuseness agrees with the extension of the sampling rates of unevenly sampled data..

3. The VLF peak observed in the R-R interval spectrum is usually missing in the ECG spectrum. This lead us to suspect that the VLF observed in the RR spectrum has its origin in aliasing.

4. In some cases the LF peak does not show up in the ECG spectrum. This led us to suspect that part of the LF peak is of aliasing origin.

5. Unlike in electronic devices, it is very difficult to devise procedures to detect aliasing in humans. In electronic devices aliasing can be easily detected by changing the sampling rate. In humans the fluctuations of the heart rate are of the same order as the required changes in the sampling rates. It will be an important task to develop a proper procedure for detecting aliasing in humans.

6. We have developed a new technique for spectral analysis for unevenly sampled data called non-uniform discrete Fourier transform (NUDFT). When employed to the RR data, below the Nyquist frequency, it gave similar results as those obtained by interpolating the data with a cubic spline. Above the Nyquist frequency, the correct peak in the spectrum was detected with great accuracy. A similar result was obtained with the recently rediscovered Lomb method. We interpret this unexpected result by a partial destruction of aliasing symmetry in both methods. More efforts should be made in order to understand the anti-aliasing properties of the above methods.

7. We consider aliasing to be a wrong symmetry, resulting from the use of an incomplete basis, which has intrinsic symmetries inconsistent with the properties of the signal. Aliasing can be partially removed by reducing the symmetry of the basis.

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Figure Captions

- Figure 1. The relative power of the ECG signal of Zahi, a) in the spectral range of 0-36 Hz, b) in the spectral range of 0-12 Hz.

- Figure 2. The relative power computed (from the ECG signal of Zahi) by four different methods, in the spectral range of 0-0.5 Hz, a) by DFT, b) by spline interpolation of the RR data, by NUDFT, d) from the ECG signal.

- Figure 3. The relative power of the ECG signal of Ori.

- Figure 4. The relative power computed (from the ECG signal of Ori) by four different methods, in the spectral range of 0-0.52 Hz. a) by DFT, b) by spline interpolation of the RR data, c) by NUDFT, d) from the ECG signal.

- Figure 5. The relative power computed (from the ECG signal of Ofek) by two different methods, in the spectral range of 0-0.6 Hz, a) by spline interpolation of the RR data, b) from the ECG signal.

- Figure 6. The relative power computed (from the ECG signal of J.C.) by two different methods, in the spectral range of 0-0.46 Hz, a) by spline interpolation of the RR data, b) from the ECG signal.

- Figure 7. The relative power computed (from the ECG signal of Ori with breathing rate of 0.04 Hz) by two different methods, in the spectral range of 0-0.62 Hz, a) by NUDFT, b) from the ECG signal.

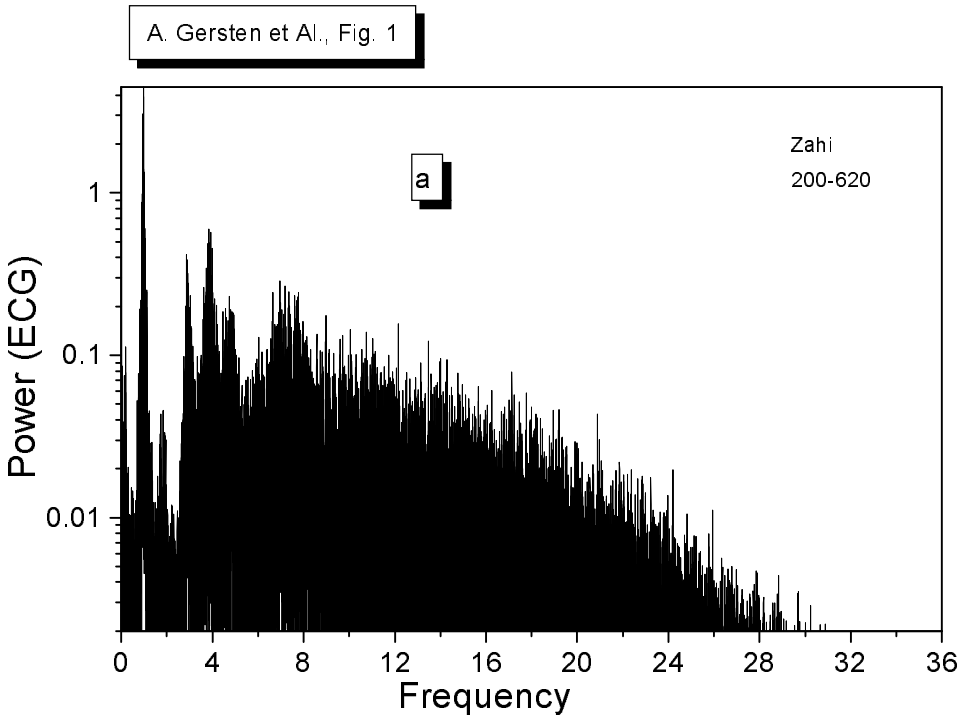
- Figure 8. The relative power of the ECG signal of Alex with a breathing rate of 1.234 Hz.

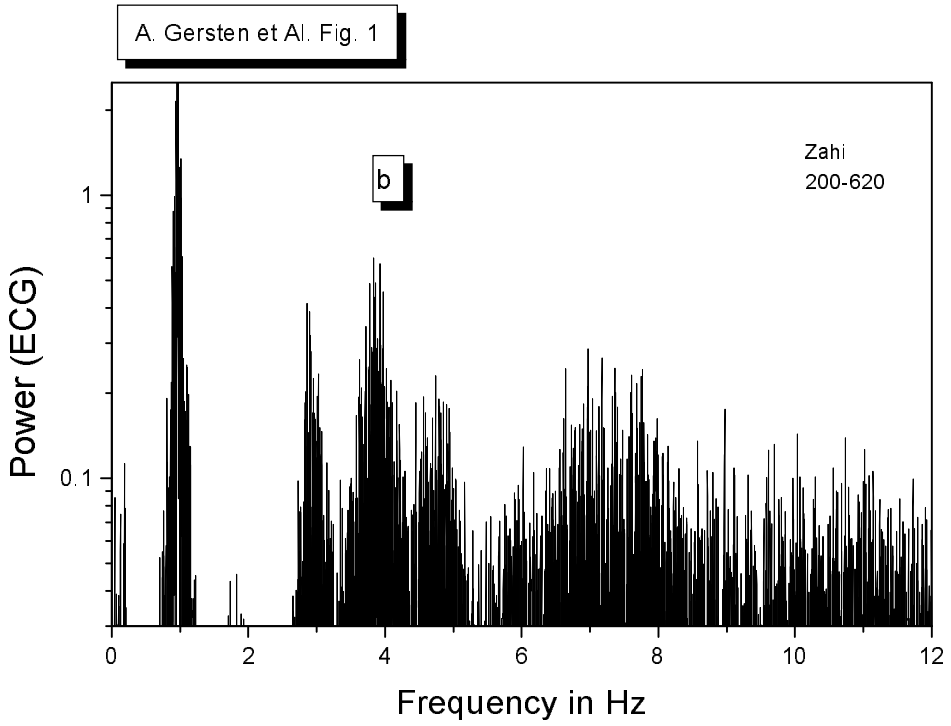
- Figure 9. The relative power computed (from the ECG signal of Alex with a breathing rate of 1.234 Hz) by two different methods, in the spectral range of 0-1.5 Hz, a) by NUDFT, b) from the ECG signal.

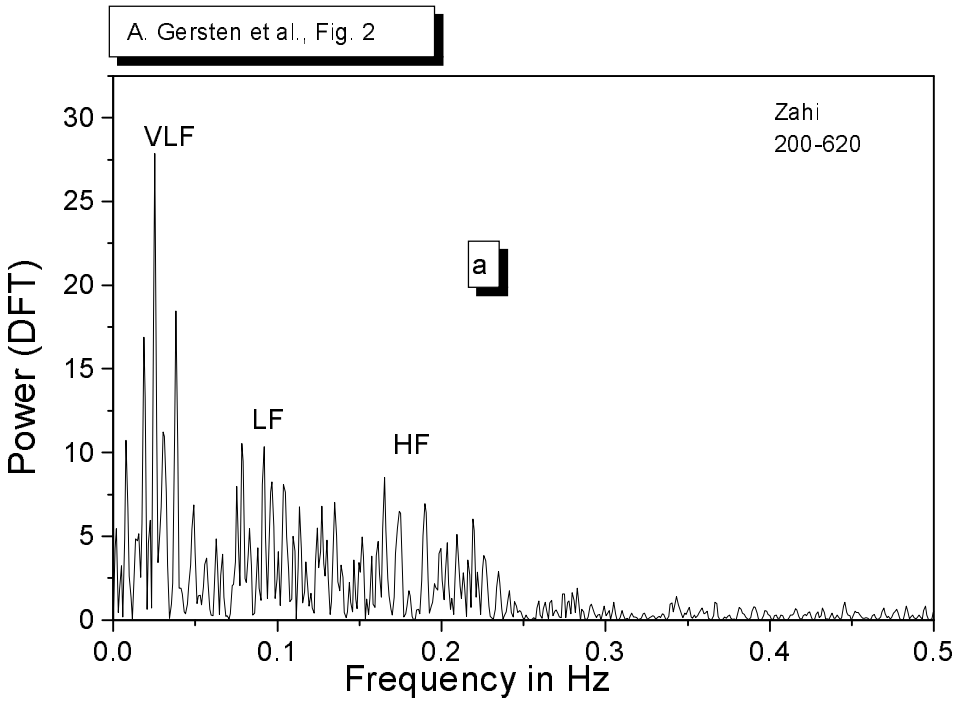
- Figure 10. A 100 bin histogram of the heart rates of Alex which are subtracted by the breathing rate of 1.234 Hz.

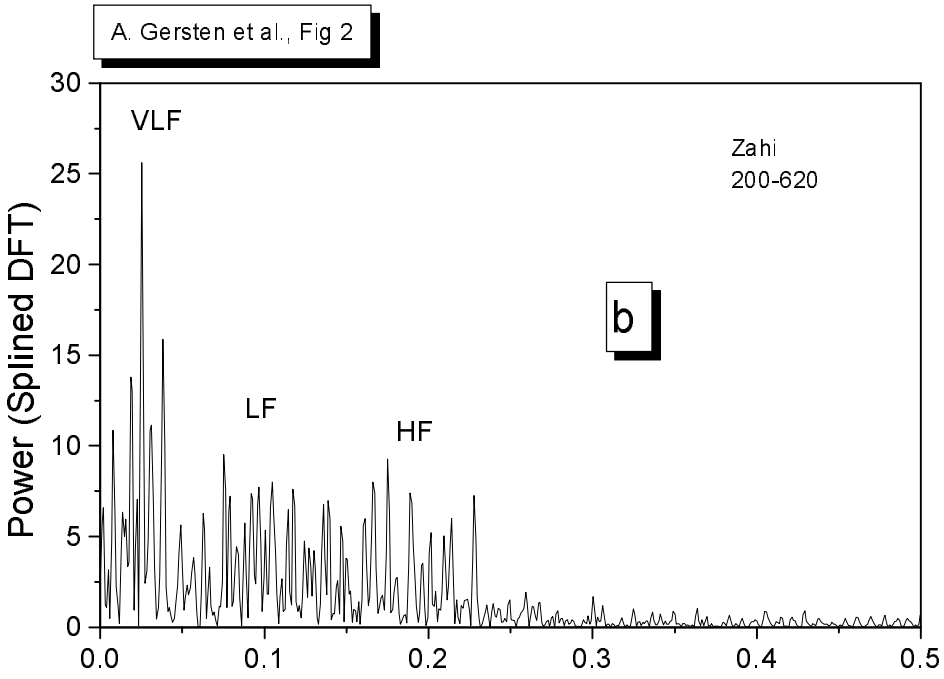
- Figure 11. The relative power of the ECG signal of Alex with a breathing rate of 1.404 Hz.

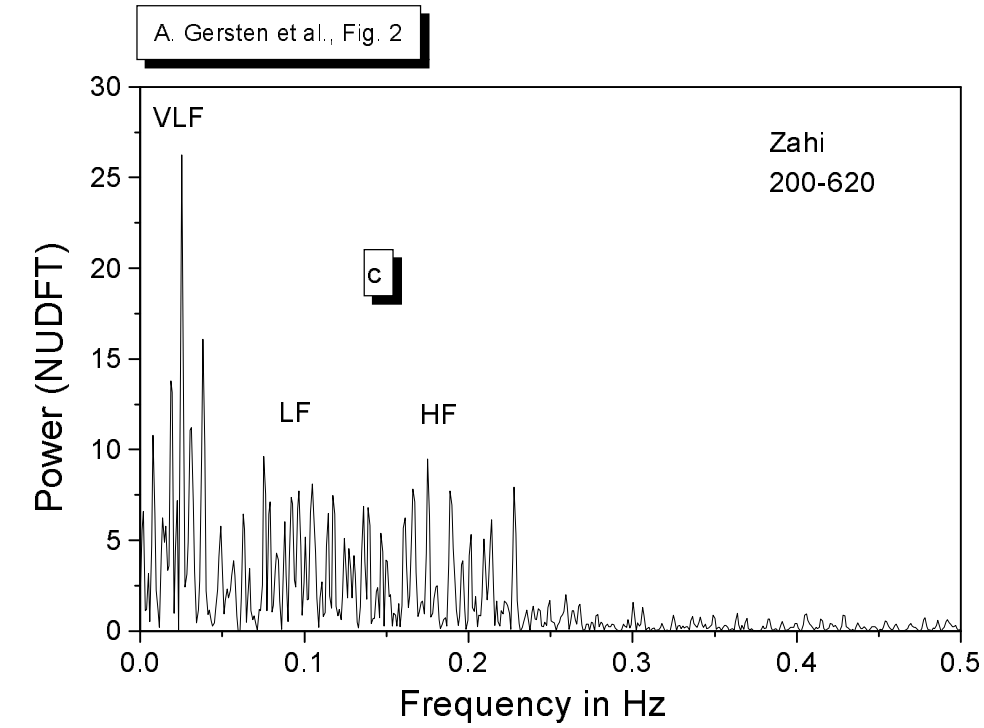
- Figure 12. The relative power computed (from the ECG signal of Alex with a breathing rate of 1.404 Hz) by two different methods, in the spectral range of 0-1.6 Hz, a) by NUDFT, b) from the ECG signal.

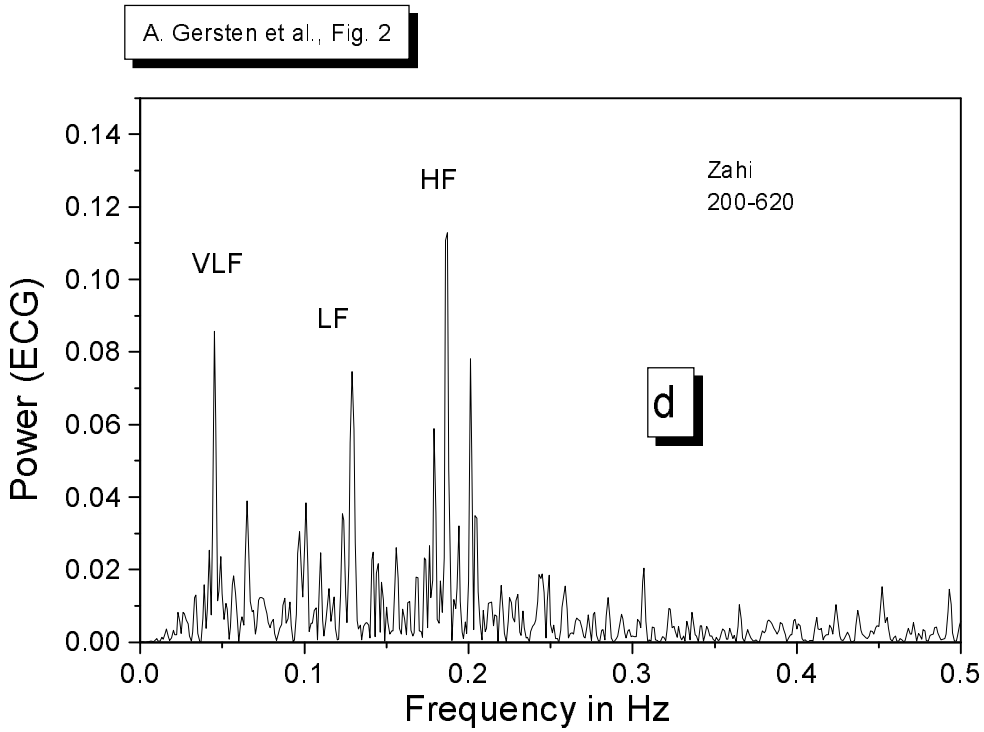

A. Gersten et Al., Fig. 1
a
Zahi
200-620
Power (ECG)
1
0.1
0.01
0
4
8
12
16
20
24
28
32
36
Frequency


A. Gersten et Al. Fig. 1
b
Zahi
200-620
Power (ECG)
1
0.1
0
2
4
6
8
10
12
Frequency in Hz

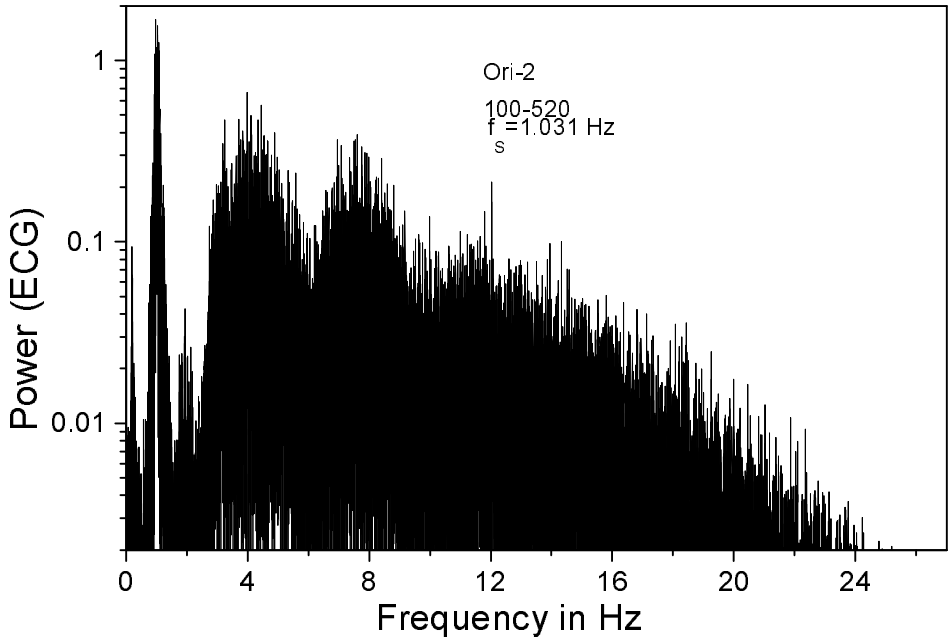

A. Gersten et al., Fig. 2
Zahi
200-620
a
VLF
LF
HF
Power (DFT)
Frequency in Hz
0
5
10
15
20
25
30
0.0
0.1
0.2
0.3
0.4
0.5


A. Gersten et al., Fig 2
Zahi
200-620
b
VLF
LF
HF
Power (Splined DFT)
0
5
10
15
20
25
30
0.0
0.1
0.2
0.3
0.4
0.5

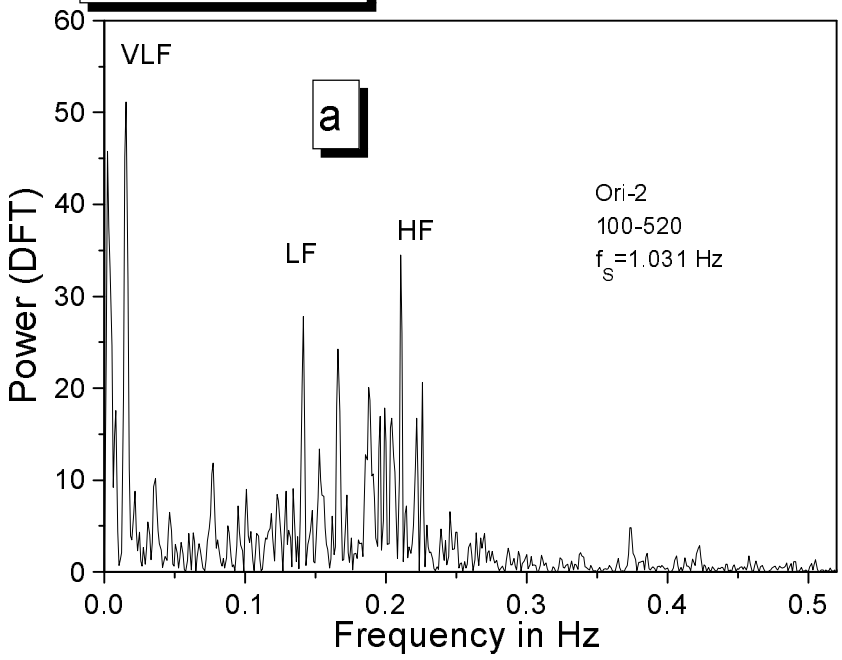

A. Gersten et al., Fig. 2
VLF
LF
HF
Zahi
200-620
c
Power (NUDFT)
Frequency in Hz
0
5
10
15
20
25
30
0.0
0.1
0.2
0.3
0.4
0.5

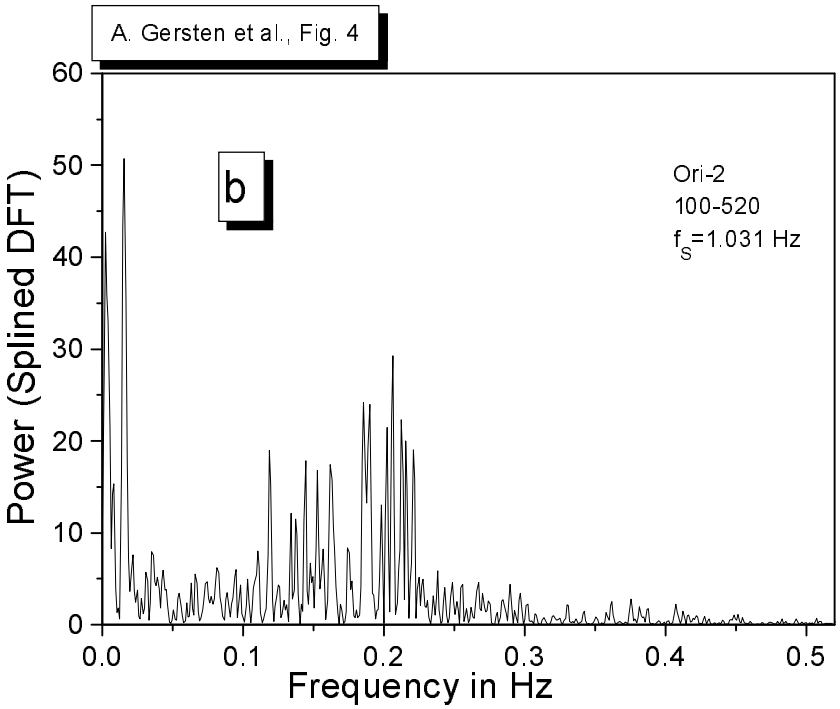

A. Gersten et al., Fig. 2
HF
VLF
LF
Zahi
200-620
d
Power (ECG)
Frequency in Hz
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.0
0.1
0.2
0.3
0.4
0.5

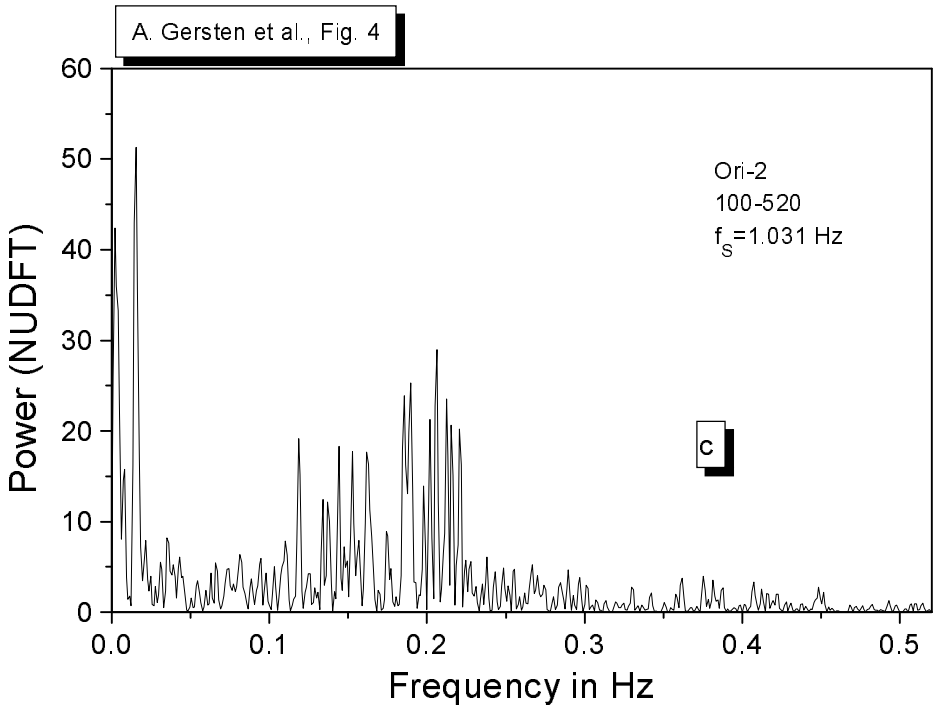
A. Gersten et al., Fig. 3

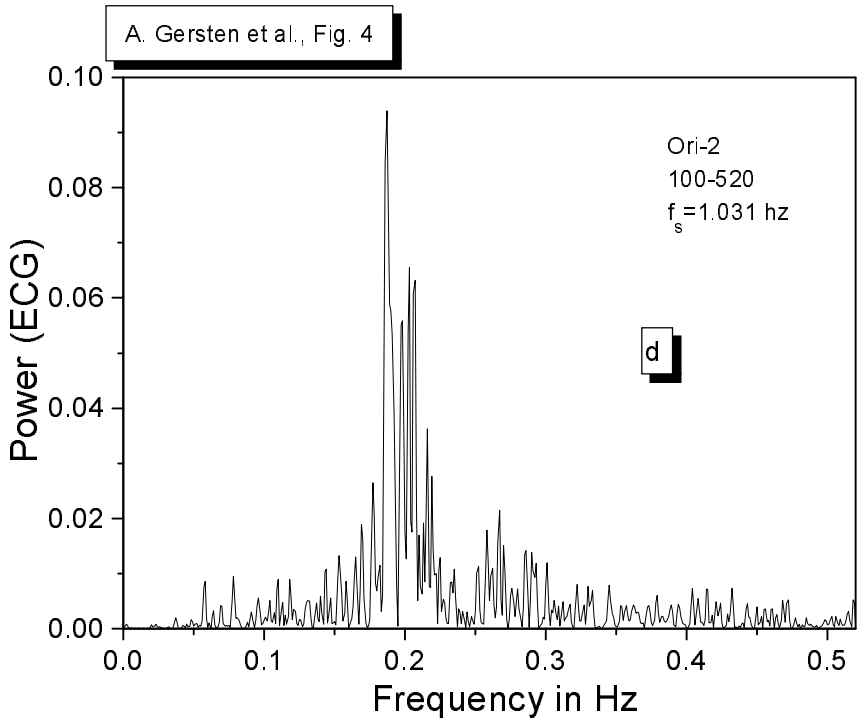


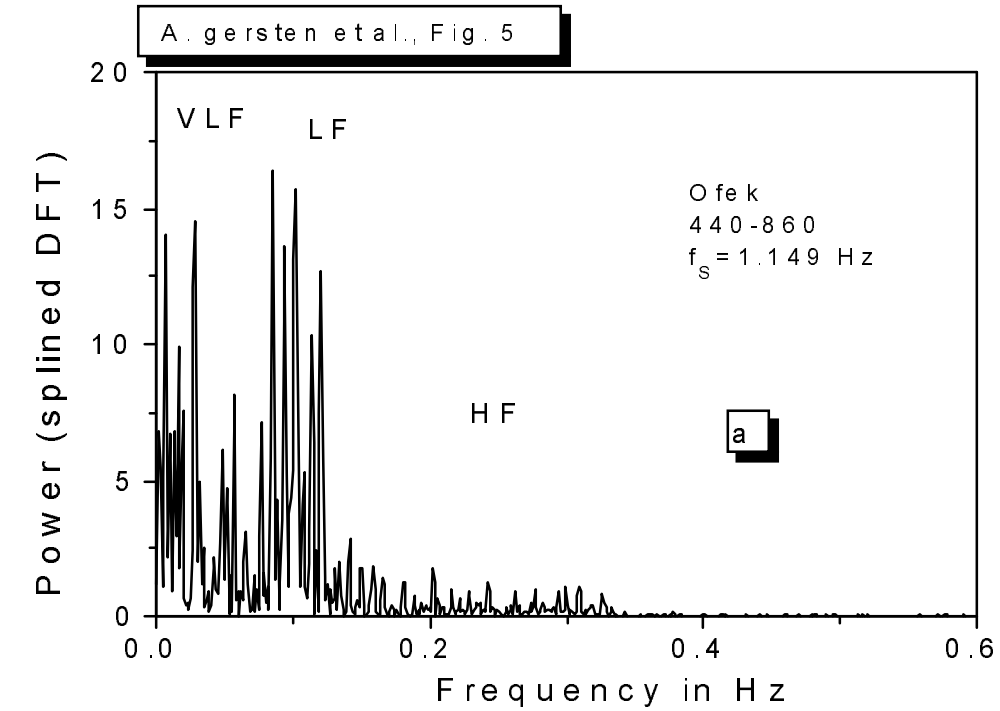
A. Gersten et al., Fig. 4



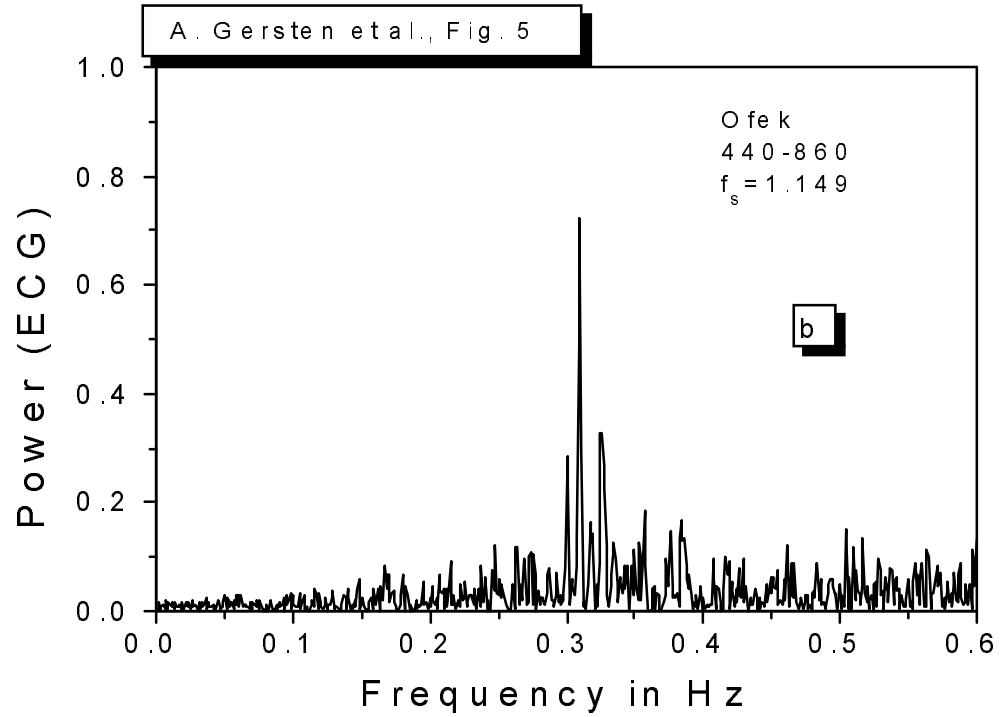

A. Gersten et al., Fig. 4
b
Ori-2
100-520
f_S=1.031 Hz
Power (Splined DFT)
Frequency in Hz
0
10
20
30
40
50
60
0.0
0.1
0.2
0.3
0.4
0.5

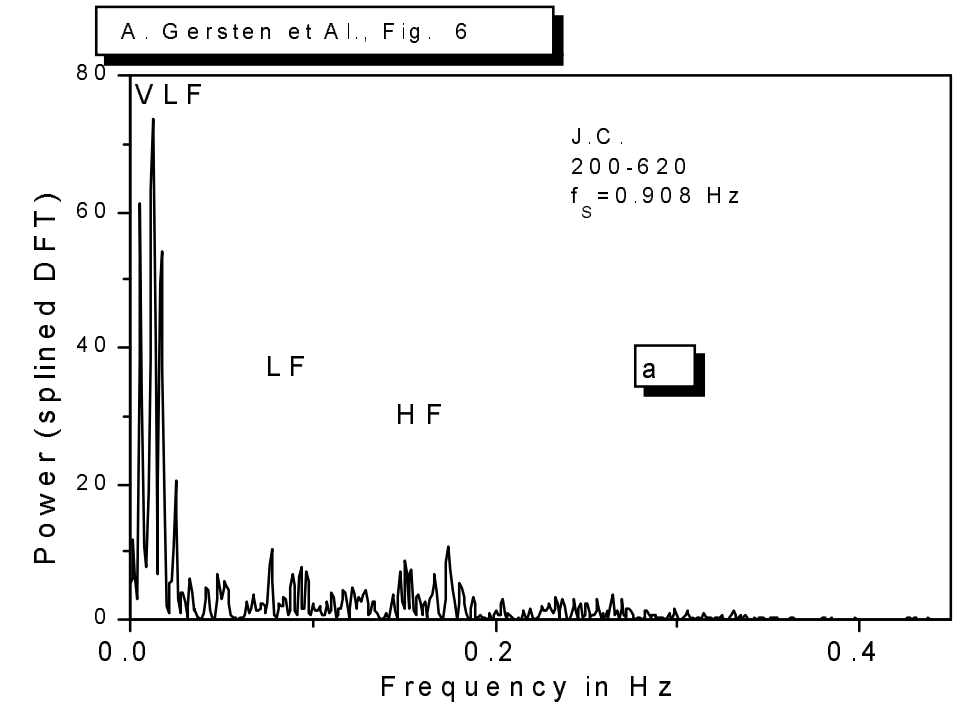

A. Gersten et al., Fig. 4
Ori-2
100-520
f_S=1.031 Hz
c
Power (NUDFT)
Frequency in Hz
0
10
20
30
40
50
60
0.0
0.1
0.2
0.3
0.4
0.5

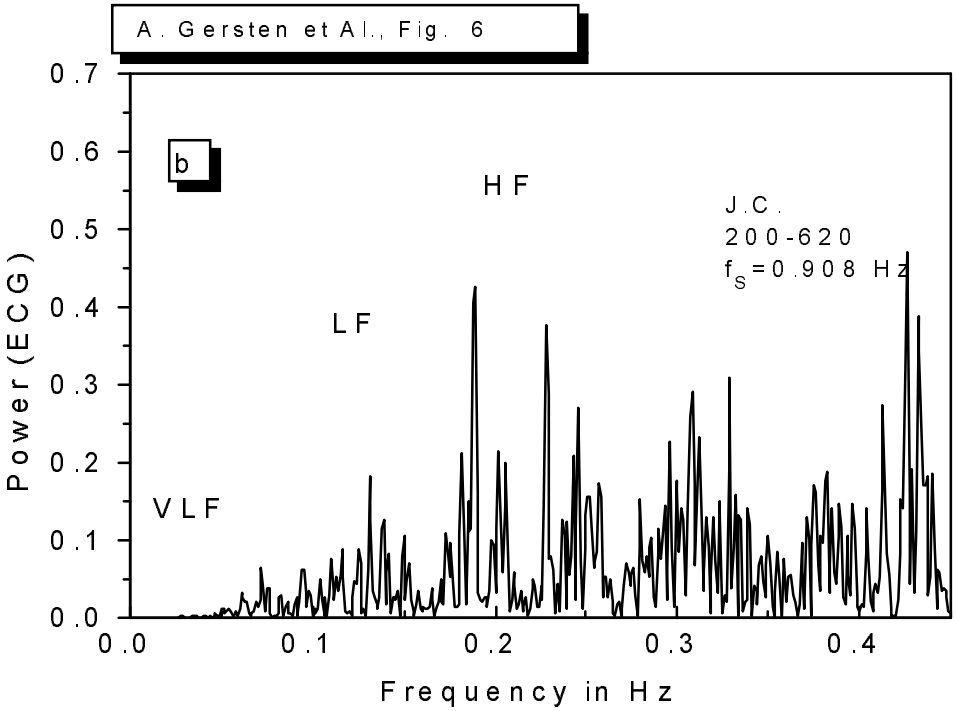

A. Gersten et al., Fig. 4
Power (ECG)
0.10
0.08
0.06
0.04
0.02
0.00
0.0
0.1
0.2
0.3
0.4
0.5
Frequency in Hz
Ori-2
100-520
f_s=1.031 hz
d

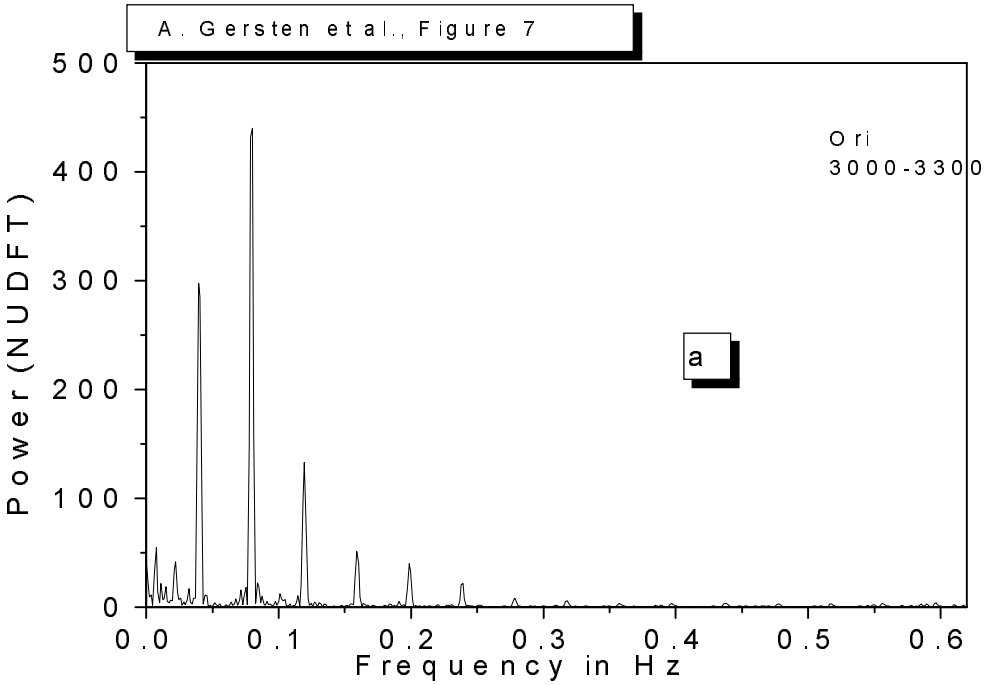

A. gersten et al., Fig. 5
VLF
LF
HF
Ofek
440-860
f_S = 1.149 Hz
a
Power (splined DFT)
Frequency in Hz
0
5
10
15
20
0.0
0.2
0.4
0.6

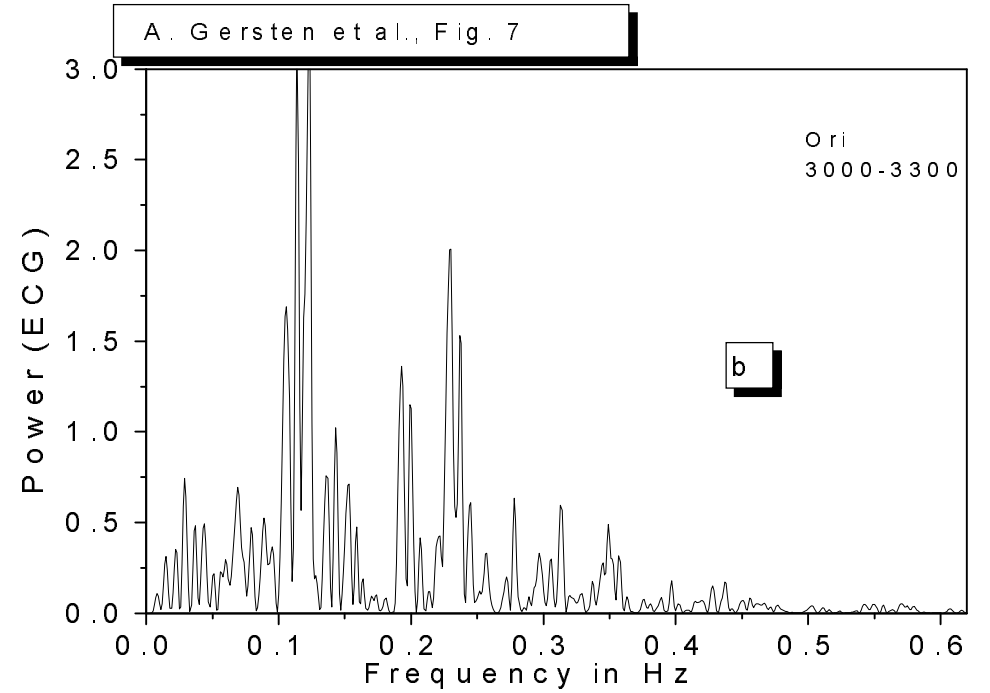
A.


A. Gersten et al., Fig. 5
Ofek
440-860
f_s = 1.149
b
Power (ECG)
Frequency in Hz
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6

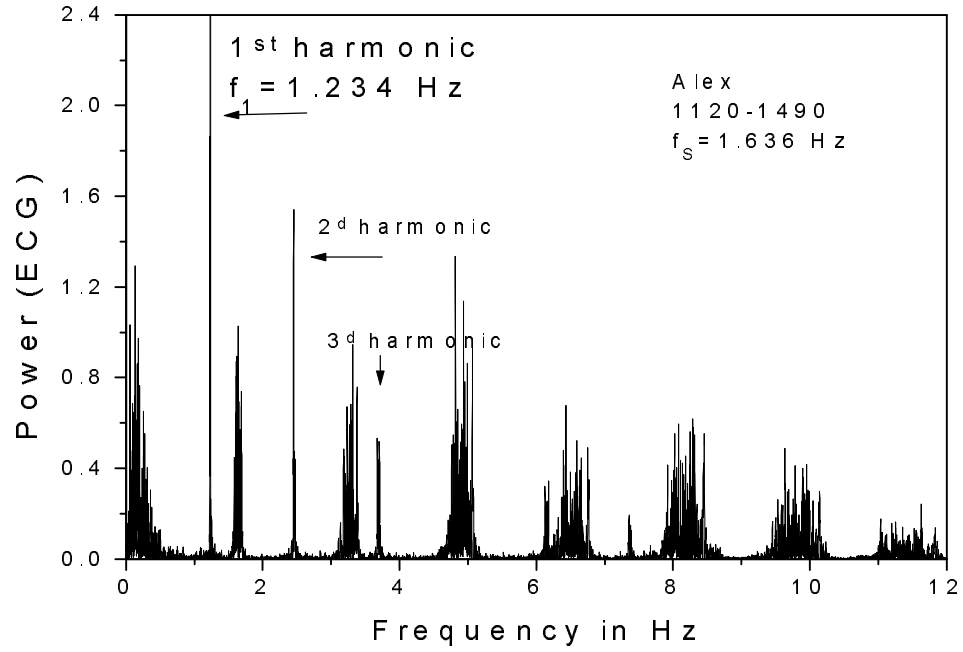

A. Gersten et Al., Fig. 6
VLF
LF
HF
J.C.
200-620
f_S=0.908 Hz
a
Power (splined DFT)
Frequency in Hz
0
20
40
60
80
0.0
0.2
0.4


A. Gersten et Al., Fig. 6
b
HF
LF
VLF
J.C.
200-620
f_S=0.908 Hz
Power (ECG)
Frequency in Hz
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.1
0.2
0.3
0.4

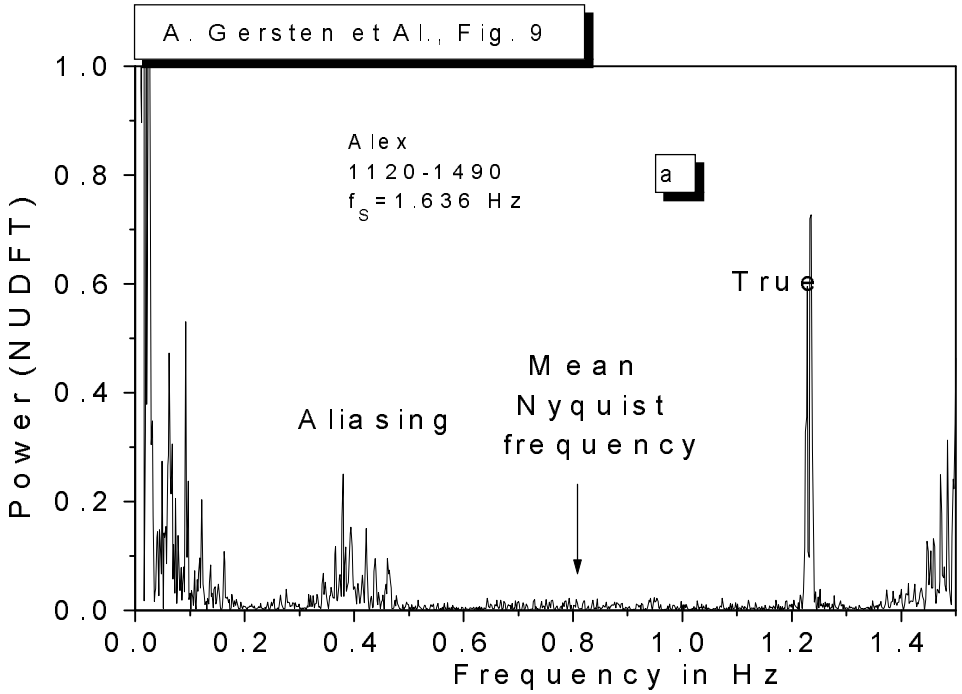

A. Gersten et al., Figure 7
Power (NUDFT)
500
400
300
200
100
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Frequency in Hz
Ori
3000-3300
a

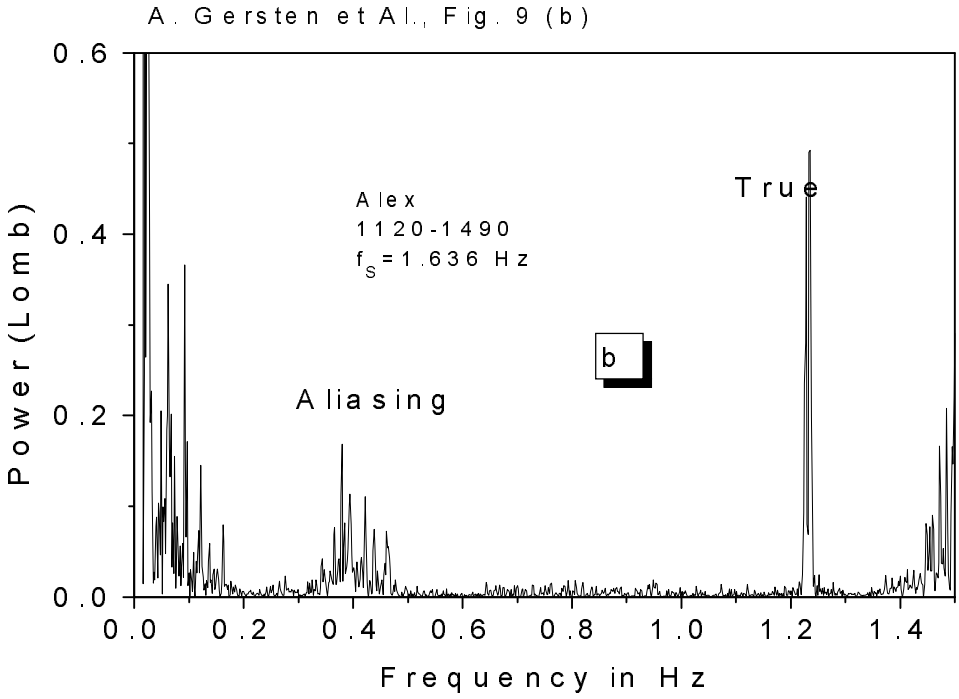

A. Gersten et al., Fig. 7
Power (ECG)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Frequency in Hz
Ori
3000-3300
b

A. Gersten et al., Figure 8

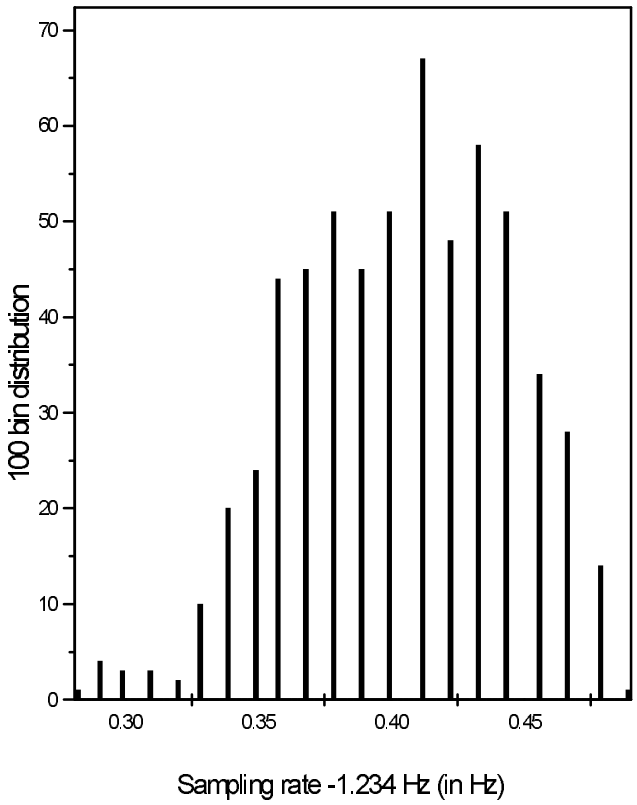


A. Gersten et Al., Fig. 9

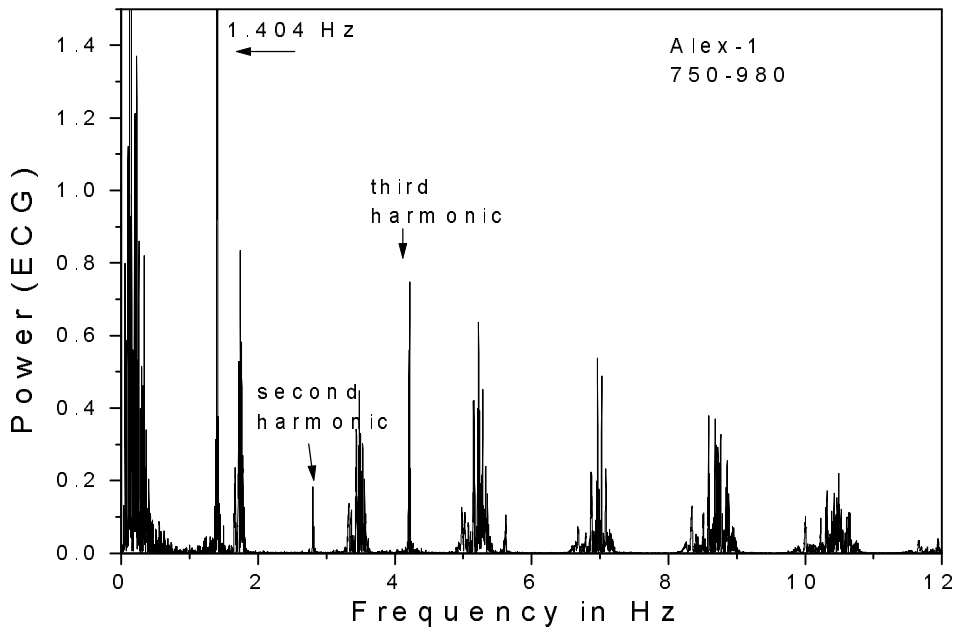


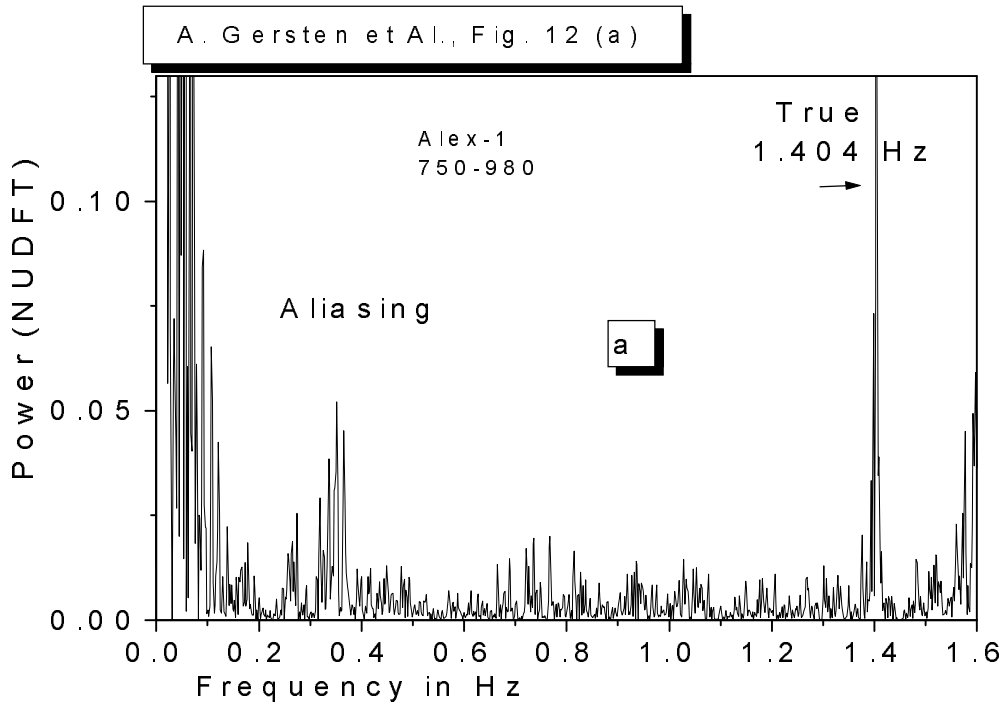

A. Gersten et Al., Fig. 9 (b)
0.6
0.4
0.2
0.0
Power (Lomb)
Alex
1120-1490
f_S = 1.636 Hz
b
True
Aliasing
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Frequency in Hz

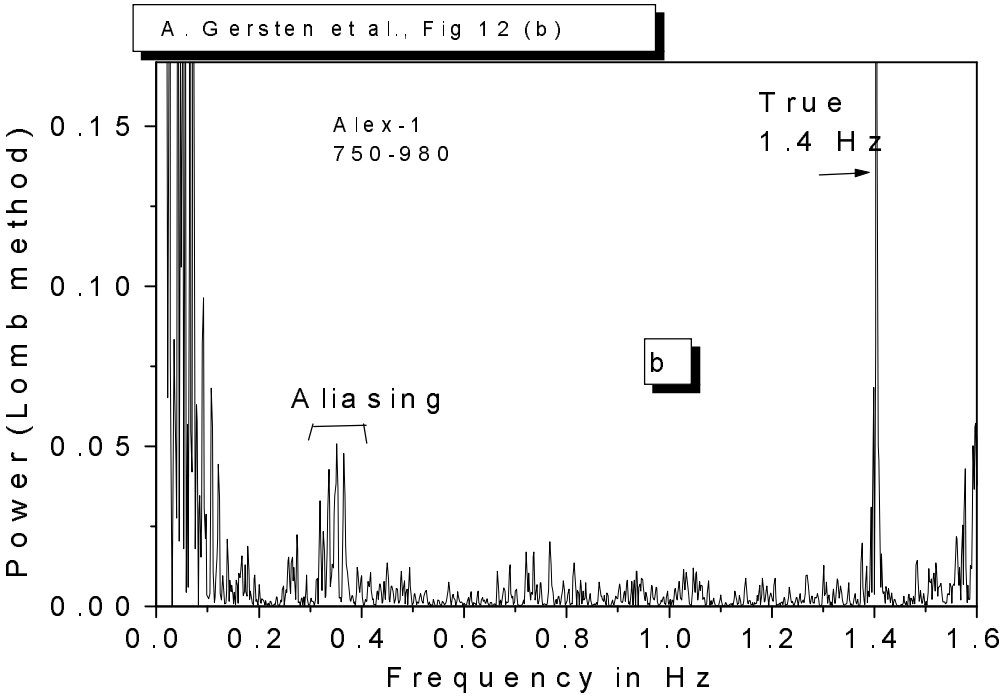
A. Gersten et Al., Figure 10



A. Gersten et al., Figure 11




A. Gersten et Al., Fig. 12 (a)
Alex-1
750-980
True
1.404 Hz
Aliasing
a
Power (NUDFT)
0.00
0.05
0.10
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Frequency in Hz


A. Gersten et al., Fig 12 (b)
Alex-1
750-980
True
1.4 Hz
Aliasing
b
Power (Lomb method)
0.00
0.05
0.10
0.15
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Frequency in Hz